Accelerating Universe in a Big Bounce Model

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Abstract

Recent observations of Type Ia supernovae provide evidence for the acceleration of our universe, which leads to the possibility that the universe is entering an inflationary epoch. We simulate it under a "big bounce" model, which contains a time variable cosmological "constant" that is derived from a higher dimension and manifests itself in 4D spacetime as dark energy. By properly choosing the two arbitrary functions contained in the model, we obtain a simple exact solution in which the evolution of the universe is divided into several stages. Before the big bounce, the universe contracts from a Λ-dominated vacuum, and after the bounce, the universe expands. In the early time after the bounce, the expansion of the universe is decelerating. In the late time after the bounce, dark energy (i.e., the variable cosmological "constant") overtakes dark matter and baryons, and the expansion enters an accelerating stage. When time tends to infinity, the contribution of dark energy tends to two third of the total energy density of the universe, qualitatively in agreement with observations.

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I. INTRODUCTION

Recently, more and more observations of Type Ia supernovae suggest that the observable universe is presently undergoing an accelerating expansion,[1,2] which is contrary to what has always been assumed that the expansion is slowing down due to gravity with a positive decelerating parameter q. This, together with the recent high precision measurement of the cosmic microwave background (CMB) fluctuations,[3,4] indicates the existence of dark energy characterized by a negative pressure, contributing with about 70%[1,2,5,6] of the totel energy density of the universe (the others being essentially dark matter and baryons). The simplest model for dark energy is the cosmological constant Λ. However, because of the so-called cosmological constant problem,[7,8] one wishes to have a dynamically decaying Λ or an evolving large-scale scalar field known as quintessence[8,9] to explain the acceleration of our universe.

It is of great interest that our conventional universe is embedded in a higher-dimensional world as required in the Kaluza-Klein theories and the brane world theories. In this paper, we consider a five-dimensional cosmological model presented by Liu and Wesson.[10] Rather than the "big bang" singularity of the standard cosmology, this 5D model is characterized by a "big bounce", at which the "size" of the universe is finite. Before the bounce the universe contracts and after the bounce the universe expands. This model is 5D Ricci-flat, implying that it is empty viewed from 5D. However, as is known from the induced matter theory,[11,12] 4D Einstein equations with matter could be recovered from 5D Kaluza-Klein equations in apparent vacuum. This approach is guaranteed by Campbell's theorem that any solution of the Einstein equations in N- dimensions can be locally embedded in a Ricci-flat manifold of (N+1)- dimensions.[13] An important result of the 5D bounce model is that a time variable cosmological "constant" can be isolated out, in a natural way, from the induced 4D energy-momentum tensor.

The 5D bounce solution contains two arbitrary functions $\mu(t)$ and $\nu(t)$. It was shown in Ref. [10] that by properly choosing $\mu(t)$ and $\nu(t)$, one can obtain exact solutions suitable to describe both the radiation-dominated universe and the matter-dominated universe as in the standard FRW models. In this paper we will show that by properly choosing $\mu(t)$ and $\nu(t)$, we can also obtain exact solutions suitable to describe our present accelerating universe which is dominated by dark energy.

II. AN ACCELERATING UNIVERSE MODEL

The 5D cosmological solution reads[10]

$$
\begin{aligned}
dS^2 &= B^2 dt^2 - A^2\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) - dy^2, \\
A^2 &= (\mu^2+k)y^2 + 2\nu y + \frac{\nu^2+K}{\mu^2+k}, \\
B &= \frac{1}{\mu}\frac{\partial A}{\partial t} \equiv \frac{\dot{A}}{\mu}.
\end{aligned}
\tag{1}
$$

Here $\mu = \mu(t)$ and $\nu = \nu(t)$ are arbitrary functions, k is the three-dimensional curvature index ($k = \pm 1, 0$), and K is a constant. This solution satisfies the 5D equations $R_{AB} = 0$ ($A, B = 0123; 5$). So one has $R = 0$ and $R^{AB}R_{AB} = 0$. The 5D Kretschmann invariant is found to be

$$I = R_{ABCD}R^{ABCD} = \frac{72K^2}{A^8}. \tag{2}$$

which shows that K determines the curvature of the five-dimensional manifold. This solution was firstly derived by Liu and Mashhoon in a different notation in Ref. [14]. The bounce property was firstly discussed by Liu and Wesson.[10] Further studies concerning the bounce singularity and other properties can be found in Ref. [15-20]. This solution can also be used to construct exact brane cosmological models.[21]

The 4D part of the empty 5D metric (1) gives an exact solution of the 4D Einstein equations with an effective or induced energy-momentum tensor. It was shown in Ref. [10] that this energy-momentum tensor could be modeled by a perfect fluid with density ρ and pressure p, plus a cosmological term Λ:

$$^{(4)}T_{\alpha\beta} = (\rho + p)u_\alpha u_\beta + (\Lambda - p)g_{\alpha\beta}. \tag{3}$$

where $u^\alpha \equiv dx^\alpha/ds$ is the 4-velocity. Furthermore, suppose the equation of state being of the form $p = \gamma\rho$, then the solution gives

$$
\begin{aligned}
\rho &= \frac{2}{1+\gamma}\left(\frac{\mu^2+k}{A^2} - \frac{\mu\dot{\mu}}{A\dot{A}}\right), \\
\Lambda &= \frac{2}{1+\gamma}\left[\left(\frac{1+3\gamma}{2}\right)\left(\frac{\mu^2+k}{A^2}\right) + \frac{\mu\dot{\mu}}{A\dot{A}}\right].
\end{aligned}
\tag{4}
$$

We can see that if the two functions $\mu(t)$ and $\nu(t)$ are given, then the two scale factors $A(t, y)$ and $B(t, y)$ are fixed by (1) and the mass density $\rho(t, y)$ and the cosmological term $\Lambda(t, y)$ are also fixed by (4). Generally speaking, on a given $y = const$ hypersurface, $\Lambda = \Lambda(t)$ is a time-variable cosmological term.

From the metric (1) we see that the form $Bdt = \left(\dot{A}/\mu\right) dt$ is invariant under an arbitrary coordinate transformation $t \to \tilde{t}(t)$. This would enable us to fix one of the two arbitrary functions $\mu(t)$ and $\nu(t)$, leaving another to account for the various content of the cosmic matter. Since Λ is a variable function, one can not simply set $\Lambda = 0$ or $\Lambda =$ constant without losing the generality of the solution. Therefore, to compare the 5D solution with the $\Lambda = 0$ FRW models, a constraint was used in Ref. [10] that *the cosmological term* Λ shown in (4) *should decay faster than the density* ρ *decreases at late-times of the universe.* Under this constraint, it was found that the choice $\mu(t) \propto t^{-1/3}$ and $\nu(t) \propto t^{1/3}$ with $k = 0$ and $K = 1$ gives a matter-dominated model, and the choice $\mu(t) \propto t^{-1/2}$ and $\nu = const$ with $k = 0$ and $K = 1$ gives a radiation-dominated model. For these two cases, the universe evolves, at late-times, with the same rate as in the FRW models.

Here we are going to compare the 5D solution with the present accelerating universe. We are aware that neither one of the two terms Λ and ρ in (4) is negligible presently. Therefore we require that Λ *should decay not faster than* ρ *does in the late-times.* In what follows we will show how this is achieved.

Astrophysical data are compatible with a $k = 0$ flat 3D space. Meanwhile, we assume the density ρ being composed mainly of cold dark matter and baryons. So we choose $k = 0$, $K = 1$ and $\gamma = 0$. Then (1) and (4) become

$$
\begin{aligned}
A^2 &= (\mu y + \frac{\nu}{\mu})^2 + \frac{1}{\mu^2}, B = \frac{\dot{A}}{\mu}, \\
\rho &= 2\left(\frac{\mu^2}{A^2} - \frac{\mu\dot{\mu}}{A\dot{A}}\right), \\
\Lambda &= \frac{\mu^2}{A^2} + 2\frac{\mu\dot{\mu}}{A\dot{A}}.
\end{aligned}
\tag{5}
$$

Here we explain ρ as the sum of the density parameters of cold dark matter and baryons, and Λ as the dark energy density. We can designate the corresponding dimensionless densities as Ω_ρ and Ω_Λ, respectively, with $\Omega_\rho + \Omega_\Lambda = 1$. Then, from (5), we obtain

$$\Omega_\rho \equiv \frac{\rho}{\rho+\Lambda} = \frac{2}{3}\left(1-\frac{\dot{\mu}A}{\mu\dot{A}}\right),$$
$$\Omega_\Lambda \equiv \frac{\Lambda}{\rho+\Lambda} = \frac{1}{3}\left(1+\frac{2\dot{\mu}A}{\mu\dot{A}}\right). \tag{6}$$

Because the 3D space is flat ($k=0$), we assume that both $\mu(t)$ and $\nu(t)$ can be expressed as power series of t. Using the constraint that Λ does not decay faster than ρ does, we consider the following choice for $\mu(t)$ and $\nu(t)$:

$$\mu(t) = t+\frac{a}{t}, \nu(t) = t^n, \tag{7}$$

where n is a parameter to be determined latter. The corresponding form of $A(t,y)$ is then given by

$$A^2 = \left[\left(t+\frac{a}{t}\right)y+\frac{t^n}{t+a/t}\right]^2+\frac{1}{(t+a/t)^2}, \tag{8}$$

If we only consider the condition $n>2$ and the late time of the universe $t \gg 1$, then (8) gives

$$A^2 = t^{2n-2}\left[1+O\left(t^{-2}\right)\right],$$
$$B = (n-1)\,t^{n-3}\left[1+O\left(t^{-2}\right)\right]. \tag{9}$$

Substituting (7) and (9) into (5) and (6), we get

$$\rho = 2\left(\frac{n-2}{n-1}\right)t^{4-2n}\left[1+O\left(t^{-2}\right)\right],$$
$$\Lambda = \left(\frac{n+1}{n-1}\right)t^{4-2n}\left[1+O\left(t^{-2}\right)\right], \tag{10}$$

and

$$\Omega_\rho = \frac{2(n-2)}{3(n-1)}\left[1+O\left(t^{-2}\right)\right], \Omega_\Lambda = \frac{n+1}{3(n-1)}\left[1+O\left(t^{-2}\right)\right]. \tag{11}$$

Thus we see that at the late time of the universe $t \gg 1$, ρ and Λ decay with the same rate. This is a very useful result in the simulation to the present accelerating universe. Remember that we have used the constraint $n>2$ in (9), (10) and (11). So if $n=2.5$,

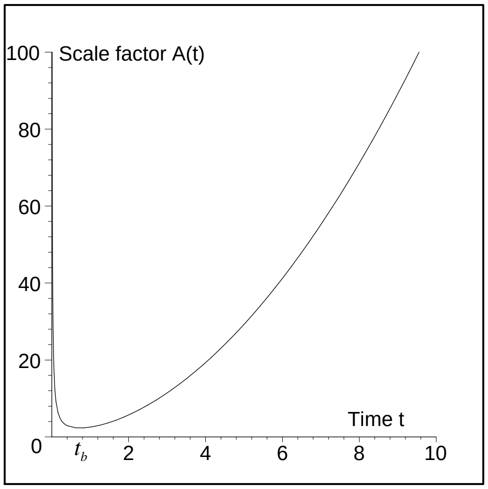

FIG. 1: Evolution of the scale factor $A(t)$ with $A^2 = [(t+1/t)+t^3(t+1/t)^{-1}]^2+(t+1/t)^{-2}$. There is a minimum at $t = t_b$ which corresponds to a bounce.

we have $\Omega_\rho \approx 2/9 \approx 0.22$ and $\Omega_\Lambda \approx 7/9 \approx 0.78$. If $n = 3$, we have $\Omega_\rho \approx 1/3 \approx 0.33$ and $\Omega_\Lambda \approx 2/3 \approx 0.67$. Current observation is about $\Omega_\rho \approx 0.3$ and $\Omega_\Lambda \approx 0.7$. So the parameter n lies between 2.5 and 3. For an illustration we let $n = 3$. Then (8) gives an exact solution being

$$A^2 = \left[\left(t + \frac{a}{t}\right) y + \frac{t^3}{t + a/t}\right]^2 + \frac{1}{(t + a/t)^2}, B = \frac{\dot{A}}{\mu}. \tag{12}$$

For $t \gg 1$, this exact solution gives

$$\begin{aligned} A &= t^2 \left[1 + O\left(t^{-2}\right)\right], B = \frac{\dot{A}}{\mu} = 2\left[1 + O\left(t^{-2}\right)\right], \\ \rho &= t^{-2}\left[1 + O\left(t^{-2}\right)\right], \Lambda = 2t^{-2}\left[1 + O\left(t^{-2}\right)\right], \\ \Omega_\rho &= \frac{1}{3}\left[1 + O\left(t^{-2}\right)\right], \Omega_\Lambda = \frac{2}{3}\left[1 + O\left(t^{-2}\right)\right]. \end{aligned} \tag{13}$$

Figure 1 is a plot of the scale factor $A(t, y)$ of the exact solution (12) on the $y = 1$ hypersurface (with $a = 1$). From this figure we can see that there is a finite minimum for the scale factor $A(t)$ at $t = t_b$ which represents a "big bounce". Before it the universe contracts, and after it the universe expands.

To see clearly how the universe evolves, we need an appropriate definition for the Hubble and the deceleration parameters. Consider the metric (1). On a given $y = const$ hypersurface the proper time can be defined as $d\tau = Bdt$. So the invariant definitions for the Hubble and

deceleration parameters should be given as[21]

$$
\begin{aligned}
H(t,y) &\equiv \frac{1}{A}\frac{dA}{d\tau} = \frac{1}{B}\frac{\dot{A}}{A} = \frac{\mu}{A}, \\
q(t,y) &\equiv -A\frac{d^2A}{d\tau^2}\Big/\left(\frac{dA}{d\tau}\right)^2 = -\frac{A\dot{\mu}}{\mu\dot{A}}.
\end{aligned} \tag{14}
$$

Then, with use of (7) and (13), equation (14) yields, for $t \gg 1$,

$$
\begin{aligned}
H &= \frac{1}{t}\left[1 + O\left(t^{-2}\right)\right], \\
q &= -\frac{1}{2}\left[1 + O\left(t^{-2}\right)\right].
\end{aligned} \tag{15}
$$

Thus we find that at the late time of the universe $t \gg 1$, the universe (12) is accelerating with $q \approx -1/2$.

Using (13) in the definition of the proper time $d\tau = Bdt$, we find

$$
\tau - \tau_b \approx 2t \tag{16}
$$

for $t \gg 1$ and $\tau \gg \tau_b$, where τ_b represents the initial bounce time. So the five-dimensional line element (1) gives

$$
dS^2 \longrightarrow d\tau^2 - \frac{1}{16}\left(\tau - \tau_b\right)^4\left(dr^2 + r^2 d\Omega^2\right) - dy^2 \tag{17}
$$

for $\tau \gg \tau_b$. This is an approximate metric to describe our present accelerating universe.

For the special exact solution (12), we can obtain an exact expression for the deceleration parameter $q(t,y)$ on a given $y = const$ hypersurface by substituting (7) and (12) in the second equation of (14). This is a long expression and we plot it in Figure 2 with $a = 1$ and $y = 1$.

Similarly, we can use the exact solutions (12) and (7) in the definition (6) to obtain exact expressions for Ω_ρ and Ω_Λ. The global evolutions of Ω_ρ and Ω_Λ are ploted in Figure 3 with $a = 1$ and $y = 1$.

From Fig. 2 and Fig. 3 we can see clearly that the global evolution of the universe is divided into four stages separated by the "big bounce" point t_b and two critical points t_{c1} and t_{c2}. The first stage is at $0 < t < t_b$ in which the universe contracts from a Λ -dominated vacuum. The bounce point t_b is a matter singularity (see also Refs. [10,15,21]), across

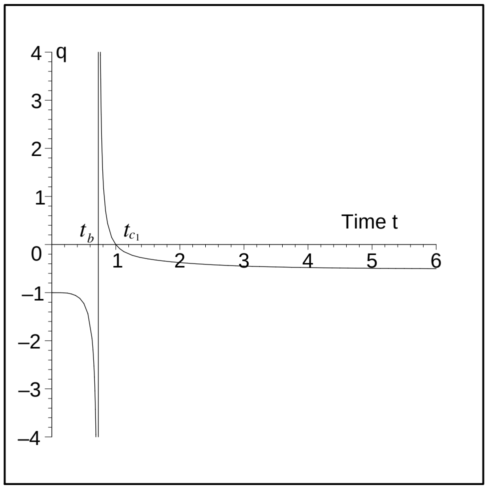

FIG. 2: Global evolution of the deceleration parameter $q(t, y)$ of the solution (12) with $a = 1$ and $y = 1$. The bouncing time is at $t = t_b$. There is a critical time t_{c1} before which the universe is decelerating and after which the universe is accelerating.

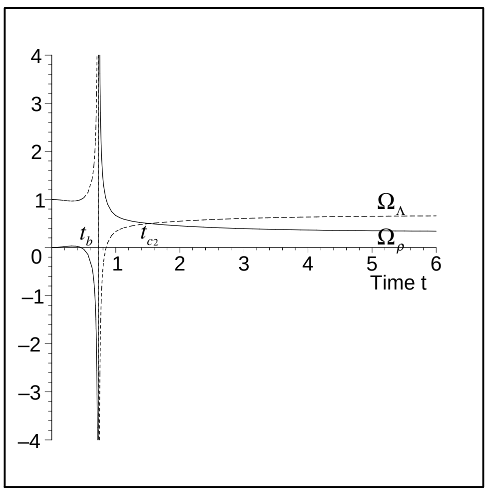

FIG. 3: Global evolutions of Ω_ρ and Ω_Λ of the solution (12) with $a = 1$ and $y = 1$. The solid line represents Ω_ρ and the dashed line represents Ω_Λ. There is a critical time t_{c2} at which Ω_Λ takes over Ω_ρ and dominates the universe.

which Ω_ρ jumps from $-\infty$ to $+\infty$ and Ω_Λ jumps from $+\infty$ to $-\infty$. The second stage is at $t_b < t < t_{c1}$ in which Ω_ρ decreases and Ω_Λ increases with $\Omega_\rho > \Omega_\Lambda$ and $q > 0$, and the universe is decelerating. The third stage is at $t_{c1} < t < t_{c2}$ in which the deceleration parameter q becomes negative, implying that the expansion of the universe turns to speed up. The fourth stage is at $t_{c2} < t$ in which $q < 0$, so the universe is accelerating. In this

stage, Ω_Λ overtakes Ω_ρ and the universe is dominated by dark energy. For $t \gg 1$ we have $\Omega_\Lambda/\Omega_\rho \longrightarrow 2$, in agreement approximately with observations of the present stage of our universe.

III. DISCUSSION

The "big bounce" solution (1) is characterized by having a "bounce", rather than a "bang", as the "beginning" of our expanding universe, and by having a evolving cosmological "constant". Mathematically, the general solution (1) contains two arbitrary functions $\mu(t)$ and $\nu(t)$. Different choices of these two functions may give different models to describe different stages of our universe. In this paper we find that a simple choice of them yields a simple exact solution (12) which can describe the present accelerating universe in a satisfactory manner.

We should emphasize that although this simple exact solution exhibits many interesting features of the bounce model, it has to be generalized to meet more observations and to explain the bouncing from a physical point of view. For instance, the two arbitrary functions $\mu(t)$ and $\nu(t)$ might be not as simple as given in (7) for a real cosmological model. Meanwhile, a variable cosmological "constant" might be also too simple to describe dark energy. It is known from literatures[8,9,22–24] that a properly chosen scalar field might be more suitable to describe dark energy. Be aware that our model (1) is 5D empty and 4D sourceful with an induced 4D energy-momentum tensor which has been supposed to contain a cosmological term Λ as shown in (3). Thus, instead of a Λ term, we probably should consider the case where the induced matter contains a scalar field as a component. We wish this scalar field may provide us with a mechanics to explain the acceleration as well as the bouncing of the universe. We are going to do this in future studies.

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